

THINKING SKILLS

9694/33

Paper 3 Problem Analysis and Solution

May/June 2019

2 hours

Additional Materials: Electronic Calculator

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.

Show your working. Marks may be awarded for correct steps towards a solution, even if the final answer is not correct. Marks may be lost if working needed to support an answer is not shown.

Calculators should be used where appropriate.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **9** printed pages, **3** blank pages and **1** Insert.

- 1 An international cycling competition is held every year in Pelatonia. Countries are invited to send a squad of cyclists to take part in the competition. There are 5 different events. The names of the events, the number of cyclists in a team for each event and the maximum number of teams allowed per squad are shown in the following table.

<i>Event</i>	<i>Number of cyclists in each team</i>	<i>Maximum number of teams allowed per squad</i>
Individual Trial	1	4
Manhattan	2	4
Chase	2	3
Derby	4	1
Road Race	6	1

For example, there are 2 cyclists in each team that takes part in the Chase and each squad is allowed to enter up to 3 teams in the Chase.

Every cyclist in a squad must take part in at least one of the events.

- (a) What is the least possible number of cyclists in a squad which enters as many teams as possible in the competition? [1]

The coach of the Keirison squad decides that none of his cyclists who takes part in the Road Race event can take part in any other event, but all other cyclists must take part in exactly two events. The squad will enter as many teams as possible.

- (b) (i) How many cyclists will be in the Keirison squad? [1]
- (ii) By labelling these cyclists as A, B, C, D, etc., show clearly one possible way in which the cyclists can be allocated to each of the 5 events. [2]

In each of the five events, a gold medal is awarded to each member of the team that finishes first; silver medals are awarded for second, and bronze for third. For example, in the Road Race, 6 gold medals, 6 silver medals and 6 bronze medals are awarded.

- (c) What is the greatest **total** number of medals, of any type, that a squad can be awarded? [1]

The squad from Graton did not have any restrictions on the number of events in which a cyclist can take part. They won exactly 7 gold medals.

- (d) (i) Show that there are three ways in which this could have been achieved. [1]
- (ii) What is the greatest number of silver medals that the Graton squad could have won? [2]

The Graton squad won as many medals as possible, given the events in which they won gold.

- (e) What is the smallest **total** number of medals that the Graton squad could have won? [2]

[Question 2 begins on the next page]

- 2 The British Diplomatic staff in Bolandia are paid a tax-free cost of living allowance to compensate for the extra costs of living abroad. This is calculated from the price of a 'basket of supermarket goods'. The annual inspection checks the local prices in three supermarkets in the capital against those in the British town Bromley (converted to Bolandian Dollars).

This year's figures for the basket were:

<i>Standard amount of</i>	<i>Bromley</i>	<i>Cotes</i>	<i>HyperFood</i>	<i>Dasamart</i>
Bread	\$27	\$26	\$29	\$30
Coffee	\$13	\$10	\$15	\$18
Shampoo	\$19	\$24	\$17	\$28
Potatoes	\$22	\$20	\$25	\$26
Macassar Oil	\$31	\$35	\$33	\$34
Cheese	\$18	\$19	\$16	\$15
Total	\$130	\$134	\$135	\$151
<i>Not included under current scheme, but also checked for possible future use:</i>				
Brill-milk	\$28	\$32	\$31	\$30

The inspectors looked for the cheapest items they could find in the local supermarkets. They determined that the basket of products is cheaper in Bolandia, so no allowance would be paid.

- (a) How much did they calculate the basket of goods would cost? [1]
- (b) It was suggested that using the average Bolandian price for each item would be fair. The mean price was calculated as \$140, but someone suggested using the median instead.

What would be the price of the basket of goods if the median were used? [1]

Since each supermarket had heavily-discounted items that distorted the comparison, it was agreed that the entire basket would be purchased in one supermarket. The cheapest total would be used instead.

- (c) Each of the three supermarkets had one of the items at half price. If these discounts had not been in place, the cheapest total would have been \$152.

Which supermarket would have had the cheapest total, and which product were they discounting? Explain why this is the only possibility. [3]

The selection of items and quantities used for the basket was set many years ago. Macassar Oil is no longer routinely purchased, so it was replaced by Brill-milk, the prices for which are shown in the table above.

- (d) Explain whether the allowance will go up, down or remain the same if the cheapest total is used but the allowance is set by
- (i) the difference between the basket price in the two countries. [1]
- (ii) the ratio of the basket prices in the two countries. [2]

The staff want the allowance to be as large as possible.

- (e) The staff noted that many of them could not afford to live in Bromley. Would it increase the allowance if the basket were calculated based on the prices in a less expensive town in Britain? Explain your answer. [1]

The staff are paid their salaries in the UK, in British Pounds. The allowance is later reduced to zero because the currency exchange rate between the two countries has changed.

- (f) What is the impact on the spending power of the staff in Bolandia? [1]

- 3 Alice runs a company that organises parties. For each party she performs the following tasks:
- Booking the room for the party
 - Sending out invitations to all of the guests
 - Providing food and drink for the party

The hotel that Alice uses for the parties that she organises has three rooms available, but each party only ever uses one room. The parties always last between 2 and 5 hours (inclusive). The details of each room are given in the table below.

<i>Room</i>	<i>Number of guests</i>	<i>Cost per hour</i>
Bijou Room	Maximum 18	\$200
Conservatory	Minimum 12 Maximum 40	\$250
Grand Ballroom	Minimum 20 Maximum 50	\$275

Alice does not organise parties for more than 50 guests.

The total cost for Alice of sending invitations and providing the food is \$24 per guest.

- (a) Alice is organising a party for 30 guests next week. The party will last for 4 hours.

What is the cheapest possible total cost for the party? [2]

When determining the price that she charges for organising a party, Alice multiplies the number of hours of the party by \$150. She then adds on an extra amount for each guest. She has decided to set the amount per guest at \$40.

- (b) What is the profit or loss that Alice would make on a 2-hour party for 10 guests? [2]

- (c) For a party lasting 3 hours, what is the smallest number of guests for which Alice would make a profit? [3]

Alice is concerned that her prices are too expensive for large parties. She has decided that she should not be making a profit of more than 20% of her costs from any party that she organises.

- (d) For the party with the smallest number of guests for which Alice would make a profit of more than 20% with her current pricing system:

(i) How many hours would the party last for? [1]

(ii) How many guests would there be? [3]

She has decided that, in future, she will charge a standard rate of \$35 for each of the first 30 guests, and a reduced rate for any further guests.

- (e) What is the lowest possible price that she can set for this reduced rate and still make a profit of at least 5% of her costs from a 3-hour party for 50 guests? [4]

[Question 4 begins on the next page]

- 4 Twenty contestants have been taking part today in *Abracadabra!*, a competition for magicians.

All the magic tricks performed in the competition are selected by the contestants themselves from a list compiled by the competition committee. Each trick has a difficulty rating of 1.0, 1.5, 2.0, 2.5, or 3.0. Every performance of a trick is given a whole number mark from 1 to 8 by five judges. The score for the performance of a trick is calculated by discarding the highest and lowest of the five marks and multiplying the sum of the other three marks by the difficulty rating.

The competition began with the qualifying round, from which the top eight advanced to the final. In the qualifying round each contestant was required to perform one trick only. Anyone who was unhappy with their score, though, had the opportunity to have a second attempt, performing either the same trick again or a different one. However, any contestant who did have a second attempt had their first score cancelled and had to accept the second score, which included a penalty subtraction of 5 points applied after the multiplication.

The following table shows the judges' marks and the scores for the qualifying performances (except Rowena's) of the eight finalists. (Marks shown are for second attempts, where applicable.)

<i>Position</i>	<i>Contestant</i>	<i>Marks</i>					<i>Score</i>
		<i>Judge 1</i>	<i>Judge 2</i>	<i>Judge 3</i>	<i>Judge 4</i>	<i>Judge 5</i>	
1st	Minerva	6	7	7	6	5	57.0
2nd	Cuthbert	8	7	8	6	7	55.0
3rd	Salazar	6	6	7	6	6	54.0
4th	Amelia	7	6	8	8	6	52.5
5th	Godric	6	6	8	6	7	52.0
6th	Helga	6	5	7	6	5	51.0
7th	Rowena	6	7	6	5	6	
8th	Kingsley	6	7	6	6	7	47.5

The final consists of three rounds. In each round the finalists perform in reverse order of their positions in the qualifying round and must perform a different trick each time. No second attempts are allowed in the final. All tricks in the last round have the judges' marks doubled before the multiplication by the difficulty rating.

Half of each finalist's qualifying score is added to the scores for the three tricks performed in the final to give the grand total. At present Salazar has just performed his third trick and is waiting for the judges' marks. The current situation is as follows:

<i>Contestant</i>	<i>Scores for:</i>			<i>Grand total</i>
	<i>First trick</i>	<i>Second trick</i>	<i>Third trick</i>	
Kingsley	51.0	57.5	102.0	234.25
Rowena	52.5	51.0	108.0	236.00
Helga	63.0	54.0	102.0	244.50
Godric	54.0	52.5	110.0	242.00
Amelia	45.0	55.0	96.0	222.25
Salazar	50.0	52.5		
Cuthbert	57.5	54.0		
Minerva	39.0	51.0		

- (a) What is the maximum score that a contestant could achieve for a trick in the qualifying round? [1]

Rowena's score is missing from the table of qualifying performances. She made a mistake during her first attempt, while performing a trick with a difficulty rating of 3.0. She chose to try the same trick again and each of the five judges awarded her 2 marks more than previously.

- (b) (i) What was her qualifying score? [2]
(ii) What was her score for the first attempt? [1]
- (c) (i) Name the contestants who qualified for the final with a trick that had a difficulty rating of 2.5. [2]
(ii) As well as Rowena, which one of the other finalists qualified with their second attempt? Justify your answer. [2]
- (d) Explain why it is not possible to deduce the difficulty rating of the first trick that Amelia performed in the final. [1]

The judges have now given their marks for Salazar's third trick: they are 7, 6, 6, 7 and 6. He knows that he has not done well enough to be in first place.

- (e) Salazar's third trick had a difficulty rating of 3.0. What is his grand total? [2]

The next finalist to perform his third trick will be Cuthbert, who has decided to perform a trick with a difficulty rating of 2.5.

- (f) (i) Show that at least 2 judges will need to award 8 marks to Cuthbert's third trick for him to finish ahead of Helga. [2]
(ii) Is it possible that Cuthbert could be certain to have won the competition upon receiving the score for his third trick? Justify your answer. [2]

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